
SOFA Vector/Matrix Library

PREFACE

The routines described here comprise the SOFA vector/matrix library. Their general appearance and coding style conforms to conventions agreed by the SOFA Review Board, and their functions, names and algorithms have been ratified by the Board. Procedures for soliciting and agreeing additions to the library are still evolving.

At present the routines are all written in Fortran 77, complying with the ANSI standard (X3.9-1978) except in two respects:

- (1) All routine names are prefixed with the string "iau_". If necessary, the string can be removed globally; the result is correctly functioning code.
- (2) All routines include an IMPLICIT NONE statement. This can be removed without affecting the behaviour of the code.

If the "iau_" string and/or the IMPLICIT NONE statements are removed globally, the resulting code is fully ANSI-compliant and is functionally unaffected.

GENERAL PRINCIPLES

The library consists mostly of routines which operate on ordinary Cartesian vectors (x,y,z) and 3x3 rotation matrices. However, there is also support for vectors which represent velocity as well as position and vectors which represent rotation instead of position. The vectors which represent both position and velocity may be considered still to have dimensions (3), but to comprise elements each of which is two numbers, representing the value itself and the time derivative. Thus:

- * "Position" or "p" vectors (or just plain 3-vectors) have dimension (3) in Fortran and [3] in C.
- * "Position/velocity" or "pv" vectors have dimensions (3,2) in Fortran and [2][3] in C.
- * "Rotation" or "r" matrices have dimensions (3,3) in Fortran and [3][3] in C. When used for rotation, they are "orthogonal"; the inverse of such a matrix is equal to the transpose. Most of the routines in this library do not assume that r-matrices are necessarily orthogonal and in fact work on any 3x3 matrix.
- * "Rotation" or "r" vectors have dimensions (3) in Fortran and [3] in C. Such vectors are a combination of the Euler axis and angle and are convertible to and from r-matrices. The direction is the axis of rotation and the magnitude is the angle of rotation, in radians. Because the amount of rotation can be scaled up and down simply by multiplying the vector by a scalar, r-vectors are useful for representing spins about an axis which is fixed.
- * The above rules mean that in terms of memory address, the three velocity components of a pv-vector follow the three position components. Application code is permitted to exploit this and all other knowledge of the internal layouts: that x, y and z appear in that order and are in a right-handed Cartesian coordinate system etc. For example, the cp function (copy a p-vector) can be used to copy the velocity component of a pv-vector (indeed, this is how the CPV routine is coded).
- * The routines provided do not completely fill the range of operations that link all the various vector and matrix options, but are confined to functions that are required by other parts of the SOFA software or which are likely to prove useful.

In addition to the vector/matrix routines, the library contains some routines related to spherical angles, including conversions to and from sexagesimal format.

Using the library requires knowledge of vector/matrix methods, spherical trigonometry, and methods of attitude representation. These topics are covered in many textbooks, including "Spacecraft Attitude Determination and Control", James R. Wertz (ed.), Astrophysics and Space Science Library, Vol. 73, D. Reidel Publishing Company, 1986.

OPERATIONS INVOLVING P-VECTORS AND R-MATRICES

Initialize

| | |
|----|---------------------------------|
| ZP | zero p-vector |
| ZR | initialize r-matrix to null |
| IR | initialize r-matrix to identity |

Copy/extend/extract

| | |
|----|---------------|
| CP | copy p-vector |
| CR | copy r-matrix |

Build rotations

| | |
|----|-------------------------|
| RX | rotate r-matrix about x |
| RY | rotate r-matrix about y |
| RZ | rotate r-matrix about z |

Spherical/Cartesian conversions

| | |
|-----|--------------------------|
| S2C | spherical to unit vector |
| C2S | unit vector to spherical |
| S2P | spherical to p-vector |
| P2S | p-vector to spherical |

Operations on vectors

| | |
|------|--|
| PPP | p-vector plus p-vector |
| PMP | p-vector minus p-vector |
| PPSP | p-vector plus scaled p-vector |
| PDP | inner (=scalar=dot) product of two p-vectors |
| PXP | outer (=vector=cross) product of two p-vectors |
| PM | modulus of p-vector |
| PN | normalize p-vector returning modulus |
| SXP | multiply p-vector by scalar |

Operations on matrices

| | |
|-----|--------------------|
| RXR | r-matrix multiply |
| TR | transpose r-matrix |

Matrix-vector products

| | |
|------|---|
| RXP | product of r-matrix and p-vector |
| TRXP | product of transpose of r-matrix and p-vector |

Separation and position-angle

| | |
|------|---|
| SEPP | angular separation from p-vectors |
| SEPS | angular separation from spherical coordinates |
| PAP | position-angle from p-vectors |
| PAS | position-angle from spherical coordinates |

Rotation vectors

| | |
|------|----------------------|
| RV2M | r-vector to r-matrix |
| RM2V | r-matrix to r-vector |

OPERATIONS INVOLVING PV-VECTORS

Initialize

ZPV zero pv-vector

Copy/extend/extract

CPV copy pv-vector
P2PV append zero velocity to p-vector
PV2P discard velocity component of pv-vector

Spherical/Cartesian conversions

S2PV spherical to pv-vector
PV2S pv-vector to spherical

Operations on vectors

PVPPV pv-vector plus pv-vector
PVMPV pv-vector minus pv-vector
PVDPV inner (=scalar=dot) product of two pv-vectors
PVXPV outer (=vector=cross) product of two pv-vectors
PVM modulus of pv-vector
SXPV multiply pv-vector by scalar
S2XPV multiply pv-vector by two scalars
PVU update pv-vector
PVUP update pv-vector discarding velocity

Matrix-vector products

RXPV product of r-matrix and pv-vector
TRXPV product of transpose of r-matrix and pv-vector

OPERATIONS ON ANGLES

ANP normalize radians to range 0 to 2pi
ANPM normalize radians to range -pi to +pi
A2TF decompose radians into hms
A2AF decompose radians into d ' "
D2TF decompose days into hms

CALLS

SUBROUTINE iau_A2AF (NDP, ANGLE, SIGN, IDMSF)
SUBROUTINE iau_A2TF (NDP, ANGLE, SIGN, IHMSF)
DOUBLE PRECISION FUNCTION
iau_ANP (A)
DOUBLE PRECISION FUNCTION
iau_ANPM (A)
SUBROUTINE iau_C2S (P, THETA, PHI)
SUBROUTINE iau_CP (P, C)
SUBROUTINE iau_CPV (PV, C)
SUBROUTINE iau_CR (R, C)
SUBROUTINE iau_D2TF (NDP, DAYS, SIGN, IHMSF)
SUBROUTINE iau_IR (R)
SUBROUTINE iau_P2PV (P, PV)
SUBROUTINE iau_P2S (P, THETA, PHI, R)
SUBROUTINE iau_PAP (A, B, THETA)
SUBROUTINE iau_PAS (AL, AP, BL, BP, THETA)
SUBROUTINE iau_PDP (A, B, ADB)
SUBROUTINE iau_PM (P, R)
SUBROUTINE iau_PMP (A, B, AMB)
SUBROUTINE iau_PN (P, R, U)
SUBROUTINE iau_PPP (A, B, APB)
SUBROUTINE iau_PPSP (A, S, B, APSB)
SUBROUTINE iau_PV2P (PV, P)
SUBROUTINE iau_PV2S (PV, THETA, PHI, R, TD, PD, RD)
SUBROUTINE iau_PVDPV (A, B, ADB)
SUBROUTINE iau_PVM (PV, R, S)
SUBROUTINE iau_PVMPV (A, B, AMB)
SUBROUTINE iau_PVPPV (A, B, APB)
SUBROUTINE iau_PVU (DT, PV, UPV)

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SUBROUTINE      iau_PVUP   ( DT, PV, P )
SUBROUTINE      iau_PVXPV  ( A, B, AXB )
SUBROUTINE      iau_PXP    ( A, B, AXB )
SUBROUTINE      iau_RM2V   ( R, P )
SUBROUTINE      iau_RV2M   ( P, R )
SUBROUTINE      iau_RX     ( PHI, R )
SUBROUTINE      iau_RXP    ( R, P, RP )
SUBROUTINE      iau_RXPV   ( R, PV, RPV )
SUBROUTINE      iau_RXR    ( A, B, ATB )
SUBROUTINE      iau_RY     ( THETA, R )
SUBROUTINE      iau_RZ     ( PSI, R )
SUBROUTINE      iau_S2C    ( THETA, PHI, C )
SUBROUTINE      iau_S2P    ( THETA, PHI, R, P )
SUBROUTINE      iau_S2PV   ( THETA, PHI, R, TD, PD, RD, PV )
SUBROUTINE      iau_S2XPV  ( S1, S2, PV )
SUBROUTINE      iau_SEPP   ( A, B, S )
SUBROUTINE      iau_SEPS   ( AL, AP, BL, BP, S )
SUBROUTINE      iau_SXP    ( S, P, SP )
SUBROUTINE      iau_SXPV   ( S, PV, SPV )
SUBROUTINE      iau_TR     ( R, RT )
SUBROUTINE      iau_TRXP   ( R, P, TRP )
SUBROUTINE      iau_TRXPV  ( R, PV, TRPV )
SUBROUTINE      iau_ZP     ( P )
SUBROUTINE      iau_ZPV    ( PV )
SUBROUTINE      iau_ZR     ( R )
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