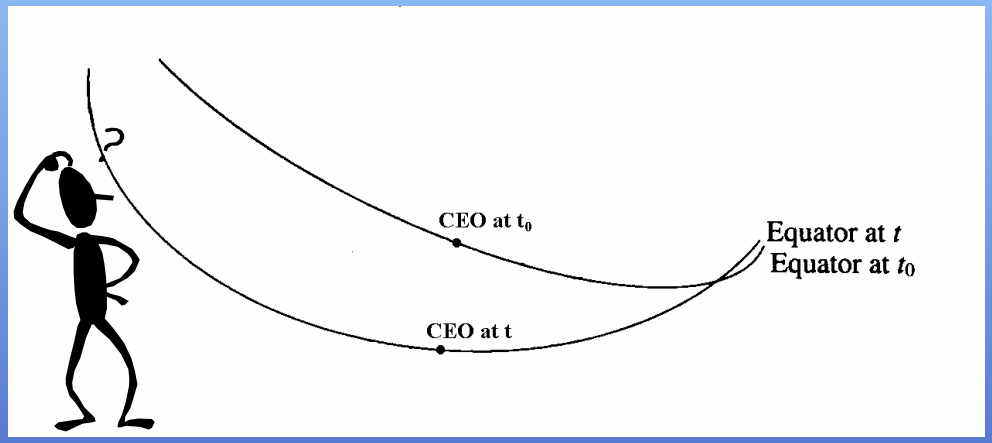
SOFA software support for IAU 2000



Patrick Wallace
Rutherford Appleton Laboratory, UK
ptw@star.rl.ac.uk

CCLRC Rutherford Appleton Laboratory

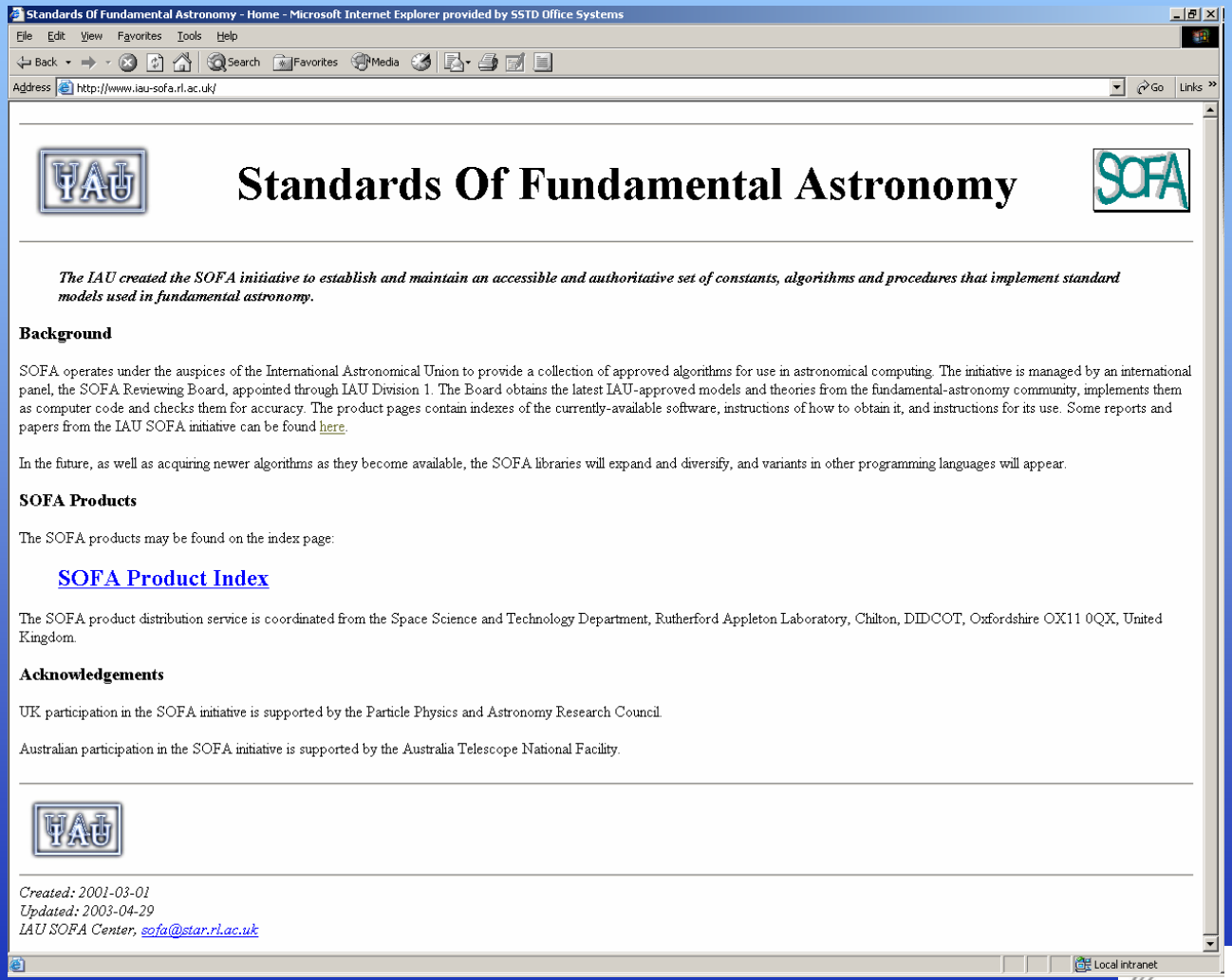
Presentation outline

- Introduction to SOFA
- IAU 2000 and SOFA
- Software design choices
- Examples

SOFA

- SOFA (Standards of Fundamental Astronomy) is an IAU initiative to provide authoritative implementations of standard algorithms.
- Controlled by the international SOFA Reviewing Board.
- Currently offers 121 Fortran subroutines:
  - Vectors and matrices
  - Calendars and time scales
  - Precession, nutation
  - Reference frames
  - etc.

www.iau-sofa.rl.ac.uk



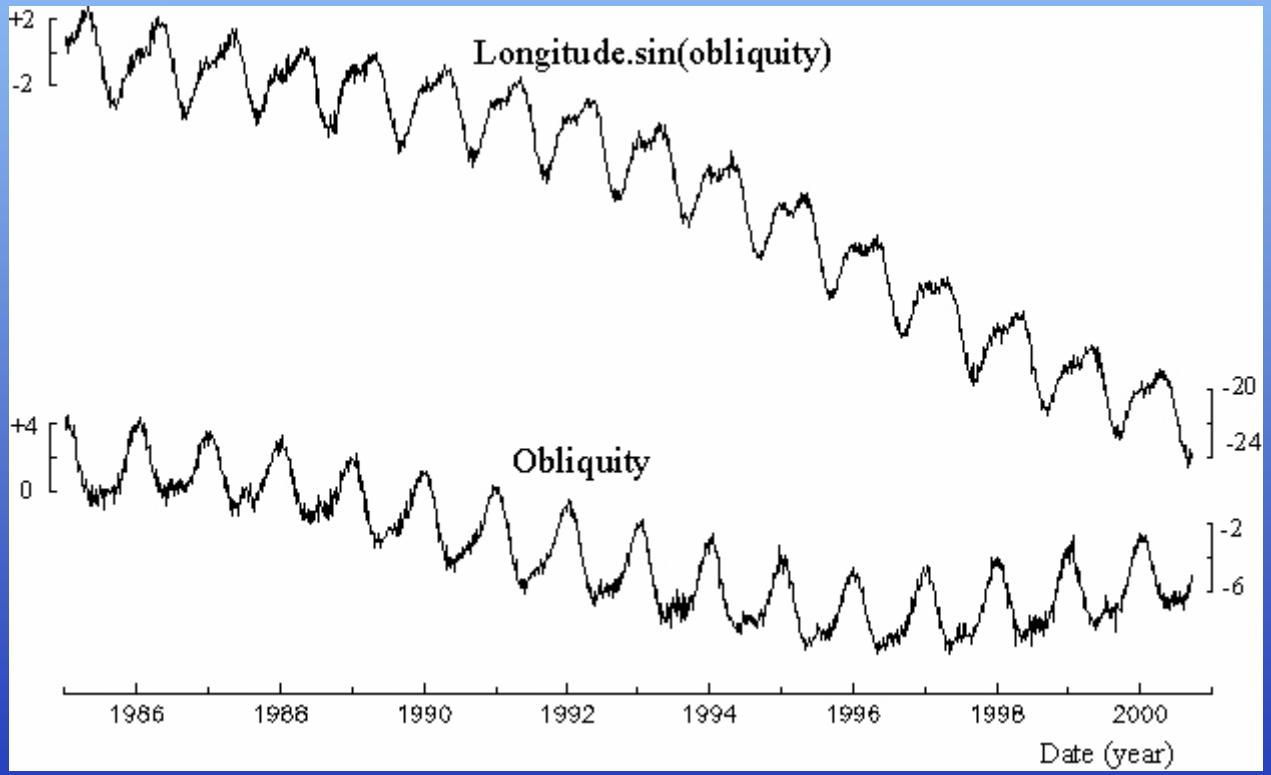
CCLRC Rutherford Appleton Laboratory

IAU 2000 inputs to SOFA software

- New, quasi-classical, model for precession-nutation of the “celestial intermediate pole”:
  - Starts with IAU 1976 precession and IAU 1980 obliquity
  - New nutation series + bias and precession corrections
  - Alternative abridged nutation model
- New way to express Earth rotation:
  - “Earth rotation angle” proportional to UT1
  - Zero point for matching RA is a “non-rotating origin”

Why there needed to be changes

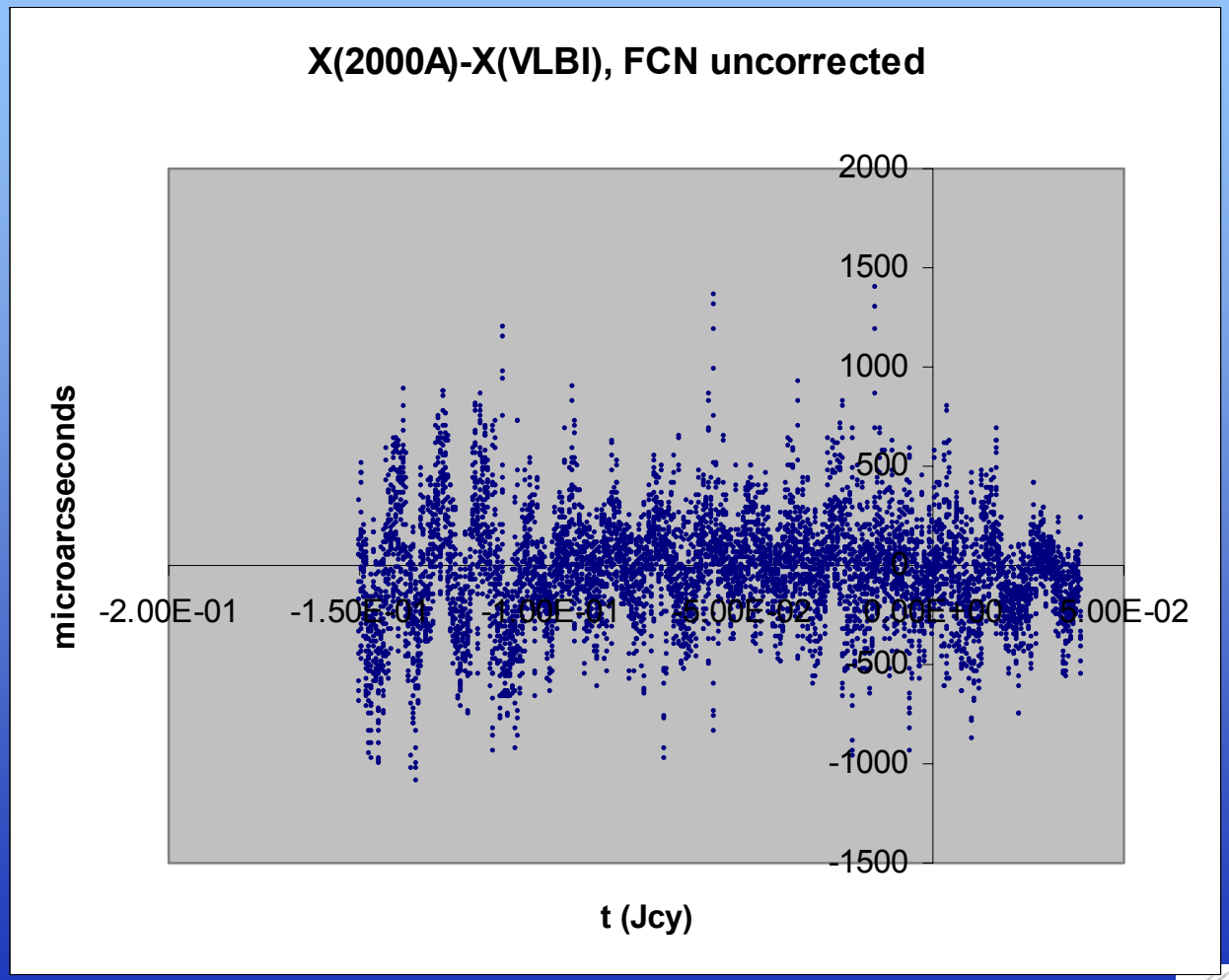
Errors in 1976/1980 precession-nutation model (mas) from VLBI



Numbers

- 1 mas ~ aberration you get from walking pace
- 1 μas ~ 30 μm at Earth's surface (~ 1 thou)
- Earth orientation known to 300 μas RMS, ~ 1 cm
- Smallest terms in nutation model ~ 1 μas
- Number of coefficients in nutation model = 2730
- Error in IAU 1976 precession ~ 1 mas/y
- Uncertainty in IAU 2000 precession ~ 30 μas/y
- Nutation-model noise floor ~ 500 μas (from FCN)

Free core nutation: the noise floor



Earth rotation, old and new

The old (classical) way:

GST_{1982} (0h) = 24110.54841 + 8640184.812866 t + 0.093104 t^2 - 6.2e-6 t^3
+ $\Delta\psi \cos \varepsilon$ + small correction terms

t is UT. The terms in red are "cross-talk" from precession-nutation.

The new (IAU 2000) way:

ERA (0h) = 24110.54841 + 8639877.317376 t

Calculating Greenwich hour angles:

For GHA = ERA-RA to work (cf. GHA = GST-RA), we simply use a different zero-point for RA, namely the CEO in place of the equinox.

Note the clean separation between Earth rotation and precession-nutation in the new system.

IAU 2000 GST expression

Former IAU 1982/94 expression:

GST_{1982} (0h) = 24110.54841 + 8640184.812866 t
+ 0.093104 t^2 - 6.2e-6 t^3
+ Δψ cos ε + small correction terms (2)

IAU 2000 compatible expression:

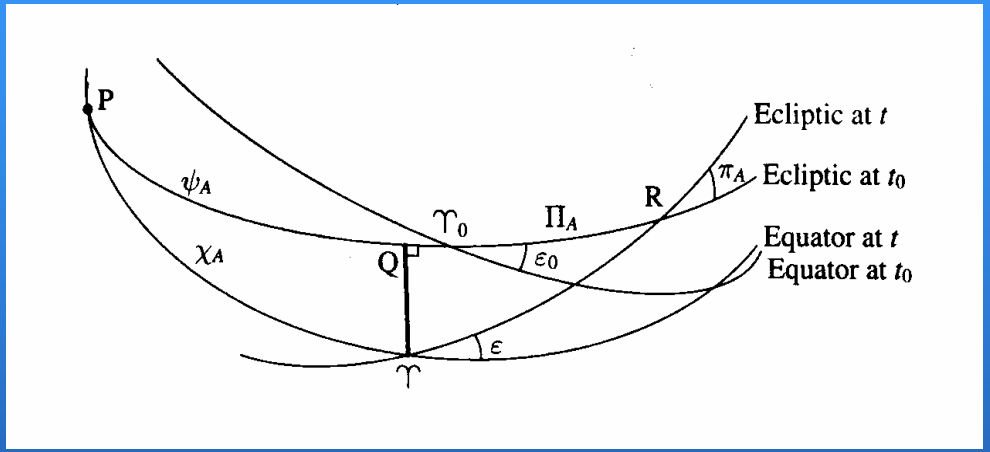
GST_{2000} (0h) = 24110.5493771 + 8639877.3173760 t_u
+ 307.4771600 t_e + 0.0931118 t_e^2
- 0.0000062 t_e^3 + 0.0000013 t_e^4
+ Δψ cos ε + small correction terms (34)

cf.

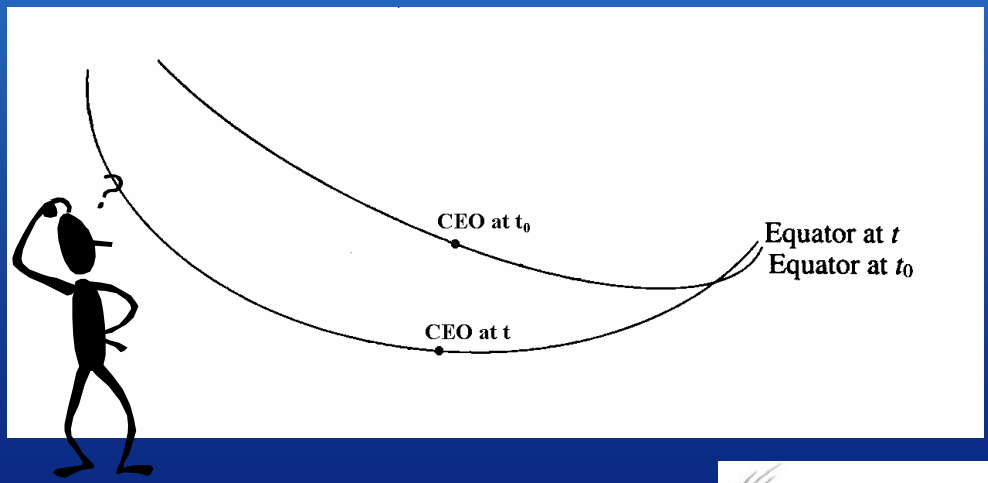
ERA (0h) = 24110.54841 + 8639877.317376 t_u

Zero points of right ascension

- Classical: zero point defined geometrically, by intersection with ecliptic. Messy (intersection of *two* moving planes) but familiar.



- New: zero point defined kinematically. Tidy but unfamiliar.



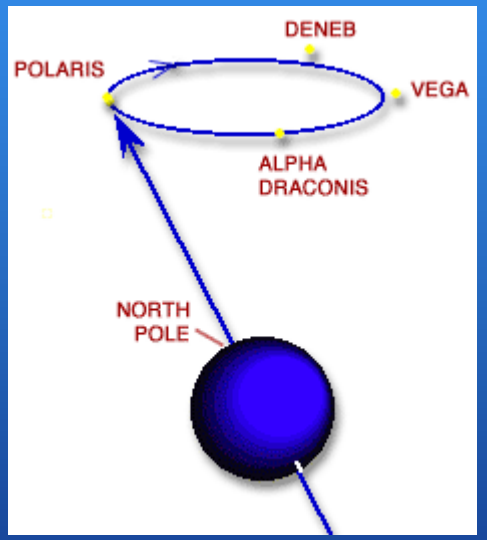
CCLRC Rutherford Appleton Laboratory

Where is the CEO?

- The CEO is merely a point on the moving celestial equator that stays as still as it can.
- Obviously, it has to move north-south in the sky as the equator precesses…
- …but it doesn't move *along* the equator: from moment to moment it moves only *at right-angles to the equator*.
- In fact it does creep along a bit, but *very* slowly.
- The CEO's present α_{ICRS} is about $00^h\ 00^m\ 00^s.0001$; by the end of the century it will have drifted only as far as $\alpha_{ICRS} = 0^h\ 00^m\ 00^s.0046$.
- The CEO can be thought of as "a kinematically defined place on the celestial equator close to where the ICRS prime meridian crosses".

What is precession?

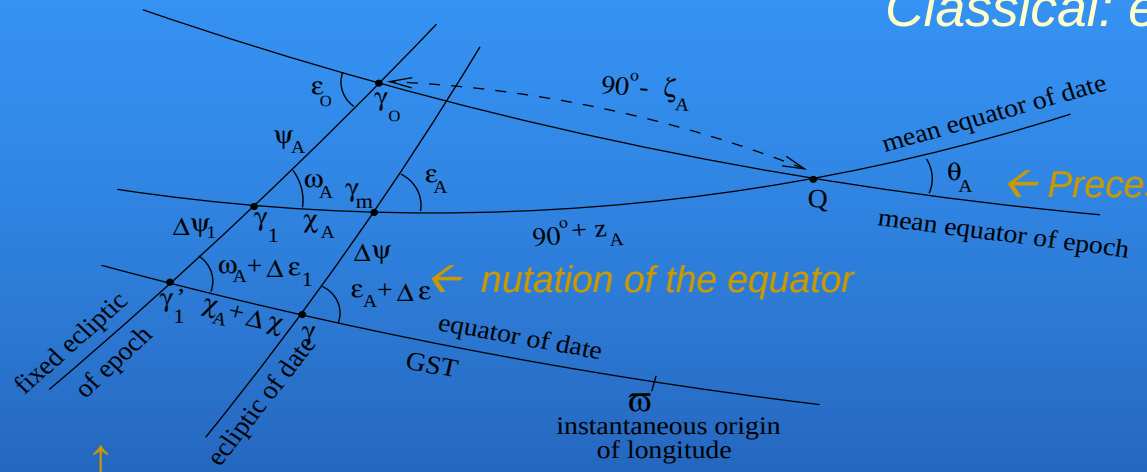
- Astronomers have traditionally talked of “the precession of the equinoxes” and also have distinguished between luni-solar precession, planetary precession and general precession.
- Lay people just talk about what the pole does.
- Indeed, precession can simply be regarded as the slow component of the motion of the Earth’s axis; this is the IAU 2000 picture.
- In IAU 2000, the clean separation between the pole’s motion and Earth rotation makes things clearer and reduces “cross-talk” effects.



CCLRC Rutherford Appleton Laboratory

Precession-nutation, old and new

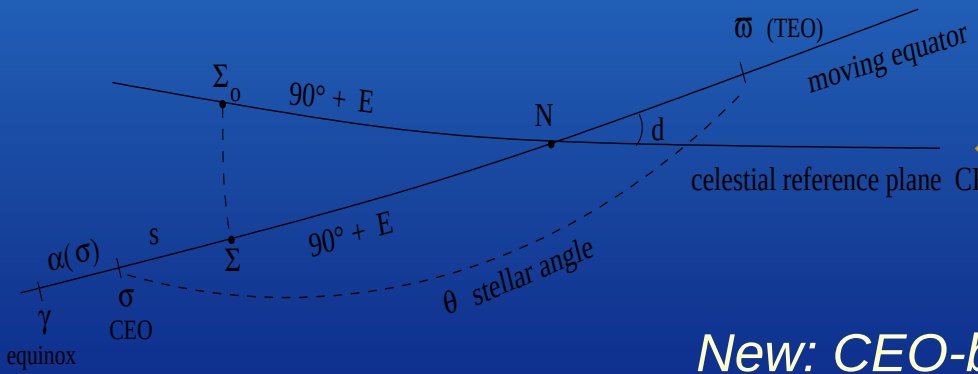
Classical: equinox-based



← Precession of the equator

← nutation of the equator

↑ Precession of the ecliptic



← Precession-nutation of the equator

New: CEO-based

CCLRC Rutherford Appleton Laboratory

So what happened to the ecliptic?

- The ecliptic remains important in a qualitative and descriptive sense…
- …and is part of constructing a precession model…
- …but is no longer needed to define the zero point of right ascension.
- The ecliptic is in any case a rather slippery concept:
  - Is the ecliptic defined by the EMB's path, or the orbital angular momentum vector? *n.b.* Difference ~ 0.1 arcsec.
  - Does it go through the Sun? Solar system barycentre? Earth-Moon-Sun barycentre?
  - What about long-period nutation terms?
- There is no "IAU 2000 ecliptic" in the SOFA software.

Precession-nutation matrix, new method

$$R(t) = R_3(-E) \cdot R_2(-d) \cdot R_3(E + s - \theta) = Q(t) \cdot R_3(-\theta)$$

X = sin d cos E, Y = sin d sin E, and Z = cos d

$$Q(t) = \begin{pmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a\left(X^2 + Y^2\right) \end{pmatrix} \bullet R_3(s)$$

where $a = \frac{1}{2} + (X^2 + Y^2)/8$

Direct models for CIP X,Y

(Capitaine, Chapront, Lambert, Wallace 2003, A&A 400)

$$X = - 0.''016617 + 2004.''191743\, t - 0.''4272190\, t^2 - 0.''1986205\, t^3 - 0.''0000460\, t^4 + 0.''0000060\, t^5 + \sum_i [(a_{s,0})_i \sin(\text{ARGUMENT}) + (a_{c,0})_i \cos(\text{ARGUMENT})] + \sum_i [(a_{s,1})_i\, t \sin(\text{ARGUMENT}) + (a_{c,1})_i\, t \cos(\text{ARGUMENT})] + \sum_i [(a_{s,2})_i\, t^2 \sin(\text{ARGUMENT}) + (a_{c,2})_i\, t^2 \cos(\text{ARGUMENT})] + \ldots$$

$$Y = - 0.''006951 - 0.''025382\, t - 22.''4072510\, t^2 + 0.''0018423\, t^3 + 0.''0011131\, t^4 + 0.''0000099\, t^5 + \sum_i [(b_{c,0})_i \cos(\text{ARGUMENT}) + (b_{s,0})_i \sin(\text{ARGUMENT})] + \sum_i [(b_{c,1})_i\, t \cos(\text{ARGUMENT}) + (b_{s,1})_i\, t \sin(\text{ARGUMENT})] + \sum_i [(b_{c,2})_i\, t^2 \cos(\text{ARGUMENT}) + (b_{s,2})_i\, t^2 \sin(\text{ARGUMENT})] + \ldots$$

precession; bias effect; nutation; cross terms precession × nutation

CCLRC Rutherford Appleton Laboratory

X,Y from classical precession-nutation matrix

$\mathbf{v}_{GCRS} = \mathbf{B} \cdot \mathbf{P} \cdot \mathbf{N} \cdot \mathbf{v}_{TRUE} = \mathbf{R} \cdot \mathbf{v}_{TRUE}$

B = frame bias matrix (GCRS → mean J2000)

$= R_3(-\Delta\alpha_0) \cdot R_2(-\Delta\xi_0) \cdot R_1(\Delta\eta_0)$

P = precession matrix

$= R_1(-\varepsilon_0) \cdot R_3(\omega_A) \cdot R_1(\psi_A) \cdot R_3(-\chi_A)$

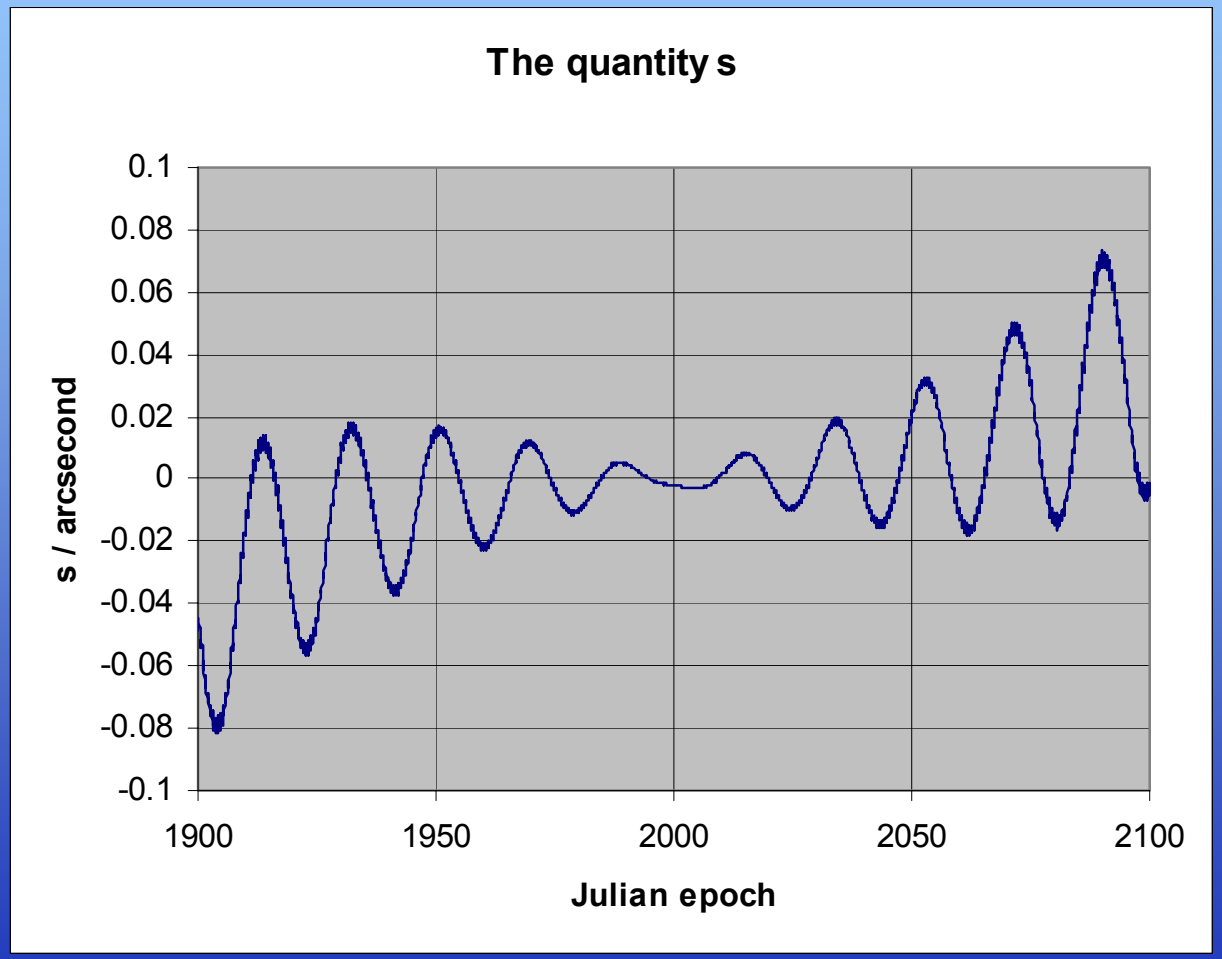
N = nutation matrix

$= R_1(-\varepsilon_A) \cdot R_2(\Delta\psi) \cdot R_1(\varepsilon_A + \Delta\varepsilon)$

R = classical precession-nutation matrix = **B·P·N**

$(X, Y)_{CIP}$ = matrix elements **R**(1,3) and **R**(2,3)

The quantity s



SOFA provision for IAU 2000

- SOFA supports both the new (CEO-based) and the classical (equinox-based) procedures.
  - The CEO-based algorithms are definitive, but the classical equivalents deliver equal accuracy for practical purposes.
- Both IAU 2000A and IAU 2000B are supported.
  - The IAU 2000B nutation series is almost as accurate (1 mas) as the full IAU 2000A series, despite being *shorter* than the IAU 1976 series.
- All these choices mean that IAU 2000 support accounts for nearly a third of the 121 SOFA routines.
  - The routines range from building blocks (e.g. compute nutation $\Delta\psi,\Delta\varepsilon$) to high-level ensembles (e.g. compute celestial-to-terrestrial matrix), enabling different simplicity/efficiency trade-offs.

Using the SOFA software

- The improved classical procedures are, as far as possible, “plug-compatible” with existing ones.
  - However, frame bias is new, and GST requires both UT and TT, so applications will need to be changed accordingly.
- You don’t have to understand the new CEO-based methods in order to benefit from the improved accuracy.
  - But for newcomers the CEO/ERA method is *simpler.*
- SOFA is Fortran-only at present.
  - Programmers in C++, Java etc. can use the SOFA routines as blueprints and benchmarks.

Other software supporting IAU 2000

IERS routines: `http://maia.usno.navy.mil/ch5subs.html`

- IAU2000A: MHB_2000 nutation
- IAU2000B: abbreviated nutation

- XYS2000A: X, Y and s
- GMST2000: GMST
- GST2000: GST
- EE2000: equation of the equinoxes
- EECT2000: complementary terms

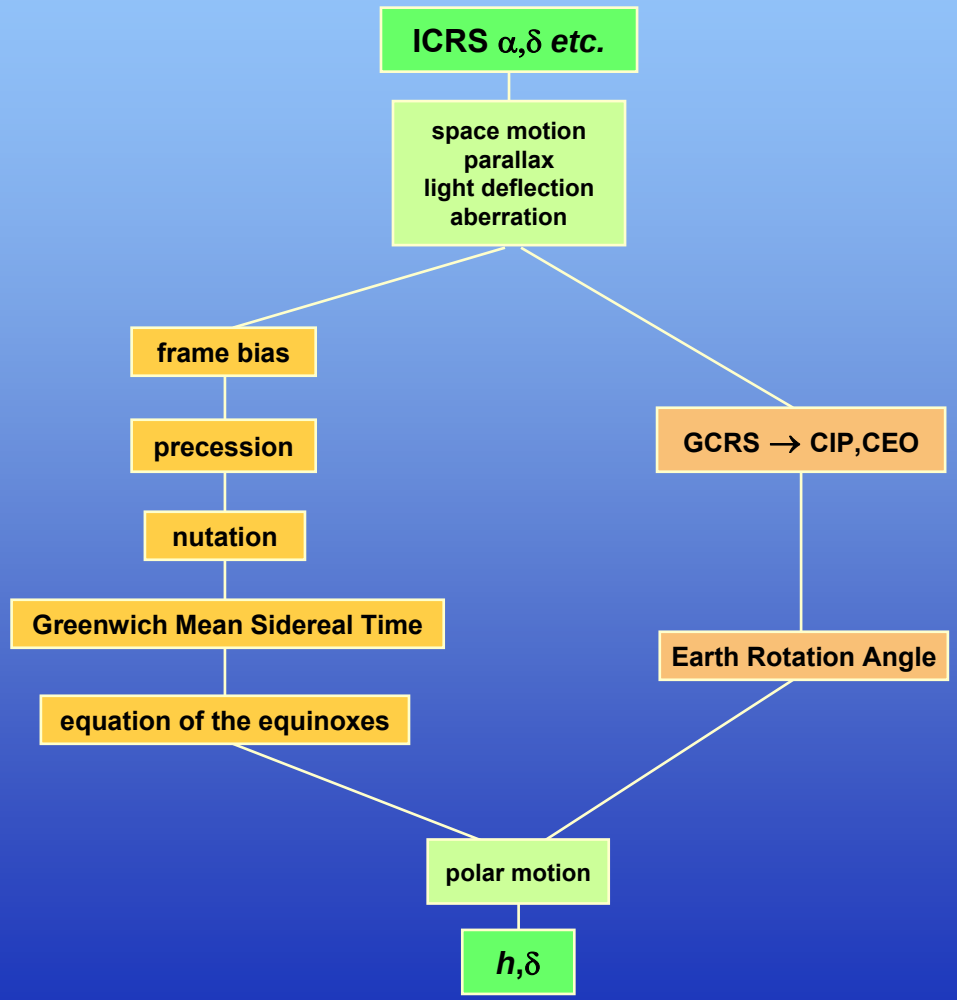
- ERA2000: Earth Rotation Angle
- SP2000: *s'*
- T2C2000: TRS-to-CRS matrix
- POM2000: polar motion matrix
- CBPN2000: classical bpn matrix
- BPN2000: new N•P•B matrix
- NU2000A: nutation, IAU 2000A
- NU2000B: nutation, IAU 2000B

In preparation:

- NOVAS
- SLALIB

CCLRC Rutherford Appleton Laboratory

Greenwich Hour Angles, old and new: methods



CCLRC Rutherford Appleton Laboratory

Greenwich hour angles, old and new: example

2004 May 31, 22h UTC
$\lambda = -104.9950$, $\varphi = +39.7427$
Fictitious star

Old-style prediction using Sidereal Time

ICRS	23 32 55.171	+52 16 38.29
Apparent RA,Dec	23 33 06.176	+52 17 43.50
Local HA,Dec	+ 8 05 50.276	+52 17 43.66

New-style prediction using Earth Rotation Angle

ICRS	23 32 55.171	+52 16 38.29
Intermediate RA,Dec	23 32 53.329	+52 17 43.50
Local HA,Dec	+ 8 05 50.276	+52 17 43.66